

Loop quantum dynamics of the gravitational collapse

Yaser Tavakoli,^{1,*} João Marto,^{1,†} and Andrea Dapor^{2,‡}

¹*Departamento de Física, Universidade da Beira Interior,
R. Marquês d'Ávila e Bolama, 6200 Covilhã, Portugal*

²*Instytut Fizyki Teoretycznej, Uniwersytet Warszawski, ul. Hoża 69, 00-681 Warsaw, Poland*

We consider a quantum description for a spherically symmetric gravitational collapse of a massless scalar field. The effective scenario from loop quantum gravity is applied to a homogeneous interior spacetime. Classical singularity that arises at the final stage of our collapsing system, is resolved and replaced by a quantum bounce. Our main purpose is to investigate the evolution of trapped surfaces during the collapse in semiclassical regime. We show that, in this regime, there exists a threshold scale below which no horizon can form as collapse evolves towards the bounce. By employing the matching conditions at the boundary shell, quantum effects are carried out to the exterior region, leading to an improved Vaidya geometry. In addition, the effective mass loss emerging in this model predicts an outward energy flux from the interior quantum geometry regime.

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I. INTRODUCTION

There are two main aspects related to the final state of gravitational collapse of a star. The first is the singularity formation, in the sense that as the radius of the star vanishes the matter energy density diverges at its centre. The second is the evolution of the horizon during the collapse. In the latter, if the trapped surfaces form as collapse proceeds, then the final singularity will be covered by the horizon and a black hole can form. Otherwise, if such trapped surfaces do not form as the collapse evolves, the radial null geodesics emerging from the singularity can reach the distant observer and hence, singularity will be naked [1–4].

It is believed that the singularity problem will be overcome in a quantum theory of gravity. Loop quantum gravity (LQG) [5–7], as a non-perturbative and background independent approach of quantum gravity, provides a fruitful ground to investigate the removal of singularities [8]. Results from the symmetry reduction of LQG (known as loop quantum cosmology (LQC)) lead to the conclusion that the spacetime singularities is resolved in quantum gravity [9]. On the other hand, the status of the classical singularities that arise at the late-time stages of the gravitational collapse, was studied in LQG [10–15]. Within this context, different matter such as a standard scalar field [10–12] or a tachyon field [13, 16] have been considered as the collapsing matter source. Therein, by employing the quantum gravity effects (such as the inverse triad correction imported from LQG), it was shown that the geometry of spacetime near the classical singularity is regular. Furthermore, some novel features, such as evaporation of horizons in the presence of quantum gravity effects was studied in Refs. [10, 12]. In

addition, it was shown in Ref. [11] that (inverse triad modifications) quantum gravity effects predict a critical threshold scale for the horizon formation which may lead to the formation of very small non-singular astrophysical black holes.

Improved LQC dynamics with a massless scalar field has been developed in the last few years [17, 18] (see also Ref. [19]). Concerning the physical implications of the singularity resolution in this model, it was shown that the classical big-bang singularity is resolved and replaced with a quantum bounce in LQC [17, 18]. In view of these elements, it is expected that the singularity arises at the end state of the gravitational collapse would be also resolved and replaced by a bounce. However, the question further arises is how loop quantum effects can indeed affect on the emergence of trapped surfaces in this model.

A trapped surface in classical general relativity is defined as a compact 2-dimensional smooth space-like submanifold of spacetime such that the families of outgoing as well as ingoing future-pointing null normal geodesics are contracting [2]. When quantum geometry becomes relevant, geodesics stop and the classical statements, based on the properties of geodesics on a differential geometry, are not valid anymore and has to be replaced by something more appropriate in quantum theory. However, in order to extract physical information out of the theory it is of interest to employ the effective theory of LQG. Quantum dynamics of LQC, including a holonomy corrections, can be approximated by an effective continuous equations of motion, which results in an effective theory of LQC [18]. The effective theory shares the form of the classical theory, but contains correction terms from the quantum theory. Furthermore, in semiclassical regime, this scenario agrees with classical general relativity. Within such a scenario, the classical statements (such as singularities and trapped surfaces), looks promising to be developed in quantum theory.

To that purpose, we apply the recent results of the effective theory of LQC in the resolution of the singular-

*Electronic address: tavakoli@ubi.pt

†Electronic address: jmarto@ubi.pt

‡Electronic address: andrea.dapor@fuw.edu.pl

ities related to the gravitational collapse of a star. More precisely, we consider in this paper, a spherically symmetric framework for gravitational collapse, whose matter content includes a scalar field, and study the physical conditions for the trapped surface formation (see also Refs. [10–12]). Then we consider the effective dynamics scenario of LQC to study the collapsing system. We investigate whether such quantum gravitational corrections can affect the dynamical evolution of the collapse and determine the formation or avoidance of the trapped surfaces.

The context of this paper is organised as follows. In section II, we provide the background scenario, by introducing the choice of spacetime geometry for the collapsing system as well as the matter source. In particular, we consider a flat Friedmann-Lemaître-Robertson-Walker (FLRW) interior spacetime to be matched to the generalised Vaidya geometry at the boundary of matter. The matter source is considered to be a homogeneous and massless scalar field. Then, we describe the model in terms of suitable variables applicable in LQG. In section III, we study the quantum gravitational collapse of our model by employing the effective dynamics scenario of LQC. In section IV, we investigate how quantum effects influence the evolution of the trapped surfaces as the collapse evolves. Furthermore, by employing the matching conditions at the boundary of the collapsing cloud, we show that the quantum effects from the semiclassical interior region are carried out to the exterior region which leads to an effective exterior geometry. Depending on the dynamics of the trapped surfaces on the interior quantum geometry, we have scenarios where the final quantum bounce is visible to a distant observer or is covered by a non-singular quantum black hole horizon. Finally, we present the conclusion of our results in section V.

II. GRAVITATIONAL COLLAPSE WITH A SCALAR FIELD

The physical model we consider is that of a collapsing spherical body (star), whose matter is described by a *homogeneous* massless scalar field¹. The matter is confined to a spherically symmetric region, whose coordinates is considered as (t, r, θ, ϕ) . In order to describe the whole spacetime structure, we assume that the spacetime is splitted into two parts: The *interior* region which constitutes the matter field; and the *exterior* region which must be matched to the interior one at the boundary with the coordinate radius $r = r_b$.

The geometry in the interior region can be modeled by a metric of the FLRW family [10, 13]:

$$g_{\mu\nu}^{\text{int}} dx^\mu dx^\nu = -dt^2 + a^2(t) (dr^2 + r^2 d\Omega^2), \quad (2.1)$$

where $d\Omega^2$ is the standard line element on the unit two sphere. We can identify every shell with its coordinate radius r . The physical radius of such a shell is given by

$$R(t, r) := a(t)r, \quad (2.2)$$

called the *area radius*. In light of the canonical analysis, which differentiates r and t , it is a good idea to fix the coordinate r and regard $R(t, r)$ as a function on the gravitational phase space.

The Einstein's equations for the interior region can be presented [4] as

$$8\pi G_0 \rho = \frac{F_{,r}}{R^2 R_{,r}}, \quad 8\pi G_0 p = -\frac{\dot{F}}{R^2 \dot{R}}, \quad \dot{R}^2 = \frac{F}{R}, \quad (2.3)$$

where G_0 is the Newton constant. A ‘dot’ and ‘,’ denote the differentiation with respect to the proper time t , and to the coordinate r , respectively. The *mass function* $F(t, r)$ is the total gravitational mass within the shell labelled by r . For a massless scalar field ϕ , the energy density ρ and the pressure p coincide, and can be expressed in terms of the matter dynamical variables as $\rho = p = \pi_\phi^2 / 2a^6$.

In order to investigate the geometry of trapped surfaces inside the star, it is convenient to study the behaviour of the radial null geodesics emerging from the interior spacetime. Let us introduce the null coordinates

$$d\xi^\pm = -\frac{1}{\sqrt{2}} [dt \mp a(t)dr]. \quad (2.4)$$

Then, the interior metric (2.1) can be recast into the double null form [20] as

$$g_{\mu\nu}^{\text{int}} dx^\mu dx^\nu = -2d\xi^+ d\xi^- + R^2 d\Omega^2. \quad (2.5)$$

The radial null geodesics are then obtained by solving $g_{\mu\nu}^{\text{int}} dx^\mu dx^\nu = 0$ with the condition $d\Omega^2 = 0$. From here we deduce that there exist two kinds of null geodesics, corresponding to $\xi^+ = \text{const.}$, and $\xi^- = \text{const.}$. We can compute the *expansion parameters* θ_\pm for these geodesics, given by [20]

$$\theta_\pm = \frac{2}{R} \partial_\pm \mathbf{R} = -\frac{2}{R} \sqrt{2} \left(\partial_t \mp \frac{\partial_r}{a(t)} \right) \mathbf{R}, \quad (2.6)$$

measuring whether the bundle of null rays normal to the sphere is diverging ($\theta_\pm > 0$) or converging ($\theta_\pm < 0$); \mathbf{R} is the Ricci scalar curvature for the metric (2.1). Introducing a new parameter $\Theta(t, r) := \theta_+ \theta_-$, we are able to say whether the spacetime is respectively, *trapped*, *untrapped* or *marginally trapped*, depending on

$$\Theta(t, r) > 0, \quad \Theta(t, r) < 0, \quad \Theta(t, r) = 0. \quad (2.7)$$

¹ This belongs to the general type I matter, which also includes dust and perfect fluids. The main property is that the energy momentum tensor admits one time-like and three space-like eigenvectors [1]: In the case at hand, they are the energy density ρ and the pressure p .

The third case in Eq. (2.7) characterises the outermost boundary of the trapped region, namely, the ‘apparent horizon’ which corresponds to the equation $\dot{R}^2 = 1$. Using Eq. (2.6) we can write $\Theta(t, r)$ as

$$\Theta = \frac{1}{2} \left(\frac{\dot{R}^2}{R^2} - \frac{1}{R^2} \right). \quad (2.8)$$

This is also to be thought of as a function of the phase space, for every fixed shell r . Specifically, we will be most interested in the boundary shell, $r = r_b$, which bounds the support of matter. In that case, we define

$$\Theta_b(t) := \Theta(t, r_b) = \frac{1}{2} \left(\frac{\dot{a}^2}{a^2} - \frac{1}{a^2 r_b^2} \right). \quad (2.9)$$

Since we are mainly interested in the trapped surfaces eventual formation due to the gravitational collapse of the interior spacetime, we assume that the star is not trapped from the initial configuration at t_i ; in other words, $\Theta(t_i, r) < 0$ for all shells $0 < r < r_b$.

To model the exterior geometry, we choose a metric of the Vaidya family². Written in advanced Eddington-Finkelstein coordinates (v, r_v, θ, ϕ) , it has the form [21]

$$g_{\mu\nu}^+ dx^\mu dx^\nu = -(1 - 2M(v)G_0/r_v) dv^2 - 2dvdr_v + r_v^2 d\Omega^2, \quad (2.10)$$

where $M(v)$ is a generic function of v , which is fixed by matching the Eq. (2.10) with Eq. (2.1) at the boundary $r = r_b$ (for a discussion, see [10, 13]). The matching condition is given by matching the area radius at the boundary Σ [4]:

$$r_v(v) \stackrel{\Sigma}{=} R(r_b, t) = r_b a(t), \quad (2.11)$$

together with the first and second fundamental forms

$$\left(\frac{dv}{dt} \right)_\Sigma = \frac{R_{,r} + r_b \dot{a}}{1 - \frac{F}{R}}, \quad (2.12)$$

$$F(t, r_b) = 2M(r_v, v)G_0, \quad (2.13)$$

$$G_0 M(r_v, v)_{,r_v} = \frac{F}{2R} + r_b^2 a \ddot{a}. \quad (2.14)$$

It should be noted that the singularity formation at $a = 0$ is independent of these matching conditions.

Matching the exterior generalised Vaidya geometry to the interior spacetime plays two important roles in a collapsing process: Firstly, it allows the matter to be radiated away as collapse evolves; and secondly, it enables the study of formation and evolution of horizon during the collapse. The second aspect is particularly important; indeed, formation of a black hole as the end state

of a collapsing star indicates that there exists a moment when an apparent horizon develops inside the cloud, and thence all the matter content collapses inside such horizon. On the other hand, if a black hole is not the end state, it means that trapped surfaces never develop at any stage of the collapse, and hence apparent horizons never form inside the star. In this paper we are mainly concerned with whether or not trapped surfaces can form in the interior region, once quantum gravity corrections are taken into account (see section III). For this reason, we will consider only the quantum correction to the space-time inside the collapsing star, however, this quantum effect can be carried by the exterior geometry through the matching conditions.

III. QUANTUM GRAVITATIONAL COLLAPSE

In this section, we discuss the quantum geometry corrections to the classical evolution presented in previous section. To do this, we need to replace the phase space variables of the collapsing spacetime with Ashtekar-Barbero variables, more suitable for LQG quantization scheme. For the symmetry reduction of the interior FLRW type (2.1), the phase space of gravity, Γ_{grav} , is two dimensional and coordinatized by the scale factor a and its conjugate momentum $\pi_a = -(6/8\pi G_0)a\dot{a}$, which satisfies the Poisson algebra $\{a, \pi_a\} = 1$. To be able to apply LQC techniques, we make a canonical transformation to a more suitable pair of variables: Let us define the oriented volume as

$$v = a^3/\alpha, \quad (3.1)$$

where $\alpha \equiv 2\pi\gamma\sqrt{\Delta}\ell_{\text{Pl}}^2$ (with $\Delta \equiv 4\sqrt{3}\pi\gamma\ell_{\text{Pl}}^2$ being the so called *area gap*, and γ is the Immirzi parameter) [9]. We easily find its conjugate momentum $b = -\gamma\sqrt{\Delta}\dot{a}/a$, with which it satisfies the Poisson algebra $\{v, b\} = 2$. Notice that, for our collapsing model $\dot{a} < 0$, so that $b > 0$. In terms of these variables, the (interior) gravitational Hamiltonian constraint can be written as $C_{\text{grav}} = -(3\pi G_0/2\alpha)|v|b^2$.

As for the (massless scalar field) matter, the Hamiltonian analysis for the homogeneous case, results in a two dimensional phase space of matter, Γ_{matt} , coordinatized by ϕ and $\pi_\phi = \alpha v \dot{\phi}$ satisfying $\{\phi, \pi_\phi\} = 1$. The matter Hamiltonian constraint is simply $C_{\text{matt}} = \pi_\phi^2/(2\alpha|v|)$. The total Hamiltonian constraint is then, given by

$$C = C_{\text{grav}} + C_{\text{matt}} = -\frac{3\pi G_0}{2\alpha}|v|b^2 + \frac{\pi_\phi^2}{2\alpha|v|}. \quad (3.2)$$

Notice that, since the scalar field ϕ does not enter in the expression of the constraint equation (3.2), its momentum π_ϕ is a constant of motion.

At this classical level, it is possible to solve the Hamilton equation for the field ϕ analytically: The equation is $\dot{\phi} = \{\phi, C\} = \pi_\phi/(\alpha|v|)$. We can also write $\dot{\phi}$

² This is a generalization of Schwarzschild metric which accounts for possible matter emissions, and realizes the astrophysical realistic case of a star surrounded by a radiating zone [21].

as $\dot{\phi} = \phi' \dot{v} = \phi' \{v, C\} = -\phi' (6\pi G_0/\alpha) |v| b$ (where a prime denotes the derivation with respect to v), which using the constraint $C = 0$, can be rewritten as $\dot{\phi} = \phi' \pi_\phi \sqrt{12\pi G_0}/\alpha$. Hence, we get

$$\frac{d\phi}{dv} = -\frac{1}{|v| \sqrt{12\pi G_0}}, \quad (3.3)$$

whose solution is

$$\phi(v) = \phi_o - \frac{1}{\sqrt{12\pi G_0}} \ln \frac{|v|}{|v_o|}. \quad (3.4)$$

Here (ϕ_o, v_o) are integration constants describing the initial conditions for the collapsing star at the $t = t_o$ space slice. We see that for $\phi \rightarrow \infty$, it is $v = 0$, i.e., the volume of the interior region vanishes; but since the matter is contained in such region, the energy density of the cloud diverges, producing a physical singularity. We will see that once quantum corrections are taken into account, the situation changes completely.

A. Loop quantization

Following Dirac program for quantization of constrained systems, we introduce a kinematical Hilbert space \mathcal{H}_{kin} mirroring at the quantum level the symplectic structure of the phase space $\Gamma = \Gamma_{\text{grav}} \times \Gamma_{\text{matt}}$, and then restrict to the kernel of the operator representing the constraint $C \approx 0$. Let us choose $\mathcal{H}_{\text{kin}} = \mathcal{H}_{\text{grav}} \otimes \mathcal{H}_{\text{matt}}$, where $\mathcal{H}_{\text{grav}} = L^2(\mathbb{R}, d\mu_{\text{Bohr}})$ and $\mathcal{H} = L^2(\mathbb{R}, d\phi)$ [9]. The main difference with respect to the canonical approach is the choice of a polymeric representation for the gravitational sector, rather than the usual Schroedinger one. Indeed, the Hilbert space of gravity is the space of square integrable functions on the *Bohr compactification of the real line*, $\bar{\mathbb{R}}$, with respect to the Haar measure $d\mu_{\text{Bohr}}$ defined on it. To our purposes, we only need to know that on $\mathcal{H}_{\text{grav}}$ the volume operator \hat{v} is well defined, and its eigenstates form a complete orthonormal basis³:

$$\hat{v}|v\rangle = v|v\rangle, \quad \langle v|v'\rangle = \delta_{v,v'}. \quad (3.5)$$

The result of using $\bar{\mathbb{R}}$ instead of \mathbb{R} is that there is no well defined operator representing b . The reason of this can be somehow found in the fundamental discreteness of space, which in turn implies that derivatives (such as \hat{b} in the standard Schroedinger representation of the algebra $\{v, b\} = 2$) do not exist, as their definition would require infinitesimal displacements. Following this interpretation, we expect no problem for finite displacements, i.e., for the exponentiated version of b . Indeed, on $\mathcal{H}_{\text{grav}}$

it is well defined the operator $\widehat{N}_\lambda := \exp(i\lambda b/2)$, where λ is a constant. So that,

$$\widehat{N}_\lambda |v\rangle = |v + \lambda\rangle. \quad (3.6)$$

As for the matter part, it is clear from our choice of Hilbert space $\mathcal{H}_{\text{matt}}$ that we use the standard Schroedinger representation, with $\hat{\phi}$ acting as multiplicative operator and $\hat{\pi}_\phi$ acting as derivative operator.

Once the action of fundamental operators is defined, any other phase space function f can be represented formally by

$$f(\widehat{v}, \widehat{b}, \widehat{\phi}, \widehat{\pi}_\phi) = f(\widehat{v}, \widehat{N}, \widehat{\phi}, \widehat{\pi}_\phi). \quad (3.7)$$

Of course, one is still faced with ordering ambiguities. As an example, we construct the operator corresponding to Θ . First, we rewrite Eq. (2.8) in terms of the new fundamental variables:

$$\Theta(t, r) = \frac{1}{2} \left(\frac{b^2}{\gamma^2 \Delta} - \frac{1}{\alpha^{2/3} r^2} \frac{1}{v^{2/3}} \right). \quad (3.8)$$

We immediately meet two problems: (i) the function b appears, which has no correspondence at the quantum level; (ii) the function $v^{-2/3}$ is non polynomial in v . In order to deal with the first problem, one observes that classically the following relation holds:

$$\frac{1}{2i} (e^{i\lambda b} - e^{-i\lambda b}) = \lambda b + O(\lambda^3). \quad (3.9)$$

So, for small values of λ , we may write $b = (e^{i\lambda b} - e^{-i\lambda b})/2i\lambda$. Now, the functions appearing on the right hand side of this expression have well defined quantum analogue, thus, we can write

$$\hat{b} = \frac{1}{2i\lambda} (\widehat{N}_\lambda^2 - \widehat{N}_\lambda^{-2}). \quad (3.10)$$

To deal with the second problem, one can use the so-called Thiemann's trick [22]: Since classically $v^{-2/3} = -(3i/\lambda) \{v^{1/3}, e^{i\lambda b/2}\} e^{-i\lambda b/2}$, at the quantum level one has

$$\widehat{v^{-2/3}} = -\frac{3}{\lambda} [\widehat{v}^{1/3}, \widehat{N}_\lambda] \widehat{N}_\lambda^{-1}. \quad (3.11)$$

Therefore, when we need the quantum correspondence of $v^{-2/3}$ in any expression, we can simply use the right hand side of the equality (3.11) instead. So, at the end of the day, the operator corresponding to Θ takes the form

$$\widehat{\Theta} = \frac{1}{2} \left[-\frac{1}{4\lambda^2 \gamma^2 \Delta} (\widehat{N}_\lambda^4 + \widehat{N}_\lambda^{-4} - 2) + \frac{3}{\lambda \alpha^{2/3} r^2} (\widehat{v}^{1/3} - \widehat{N}_\lambda \widehat{v}^{1/3} \widehat{N}_\lambda^{-1}) \right]. \quad (3.12)$$

The most important operator to be implemented on \mathcal{H}_{kin} is the Hamiltonian constraint (3.2). Indeed, the final step in Dirac quantization is to restrict our attention to the

³ Notice that the orthonormality is with respect to the Kronecker delta, $\delta_{v,v'}$.

states belonging to $\text{Ker}\widehat{C}$, since they are the *physical states*. Looking at its classical form, Eq. (3.2), we again see the same problems that attacked $\widehat{\Theta}$. Despite the fact that we have a closed form for this operator, we will not be using it in this paper. Indeed, our propose herein this paper is to study the dynamics of the trapped surfaces evolving during the collapse by employing the classical statements⁴. To this aim, it is sufficient to study our system within the semiclassical regime, by resorting to the “effective dynamics” procedure [18]. In other words, we will forget about the fundamental quantum theory, and consider only the first order modifications to the classical theory, thereby obtaining ‘effective Hamiltonian’ equations of motion [23].

B. Effective theory

A well defined treatment exists, based on the existence of a “minimal loop” around which the holonomy is computed, and used as the best approximation for b . However, for our purposes, it is enough to adopt the “effective dynamics” methods. Since we are interested in an effective classical theory, we will not turn C into an operator, but rather consider the classical modifications that would mimic the quantum behaviour at first order. The only such a modification we will take into account is the so-called *holonomy correction*, which consists in replacing b with the phase space function derived by the classical observation that

$$\frac{1}{2i\lambda} (e^{i\lambda b} - e^{-i\lambda b}) = \frac{\sin(\lambda b)}{\lambda}, \quad (3.13)$$

obviously inspired by Eq. (3.10). Since this is the only change, it is expected that the classical theory is recovered for small b , i.e., as long as the Hubble parameter, \dot{a}/a is small; in other words, the difference in the dynamics of observables will be non-negligible only in the vicinity of the classical singularity.

Plugging the substitution (3.13) in Eq. (3.2), we get the effective Hamiltonian [18, 23]:

$$C_{\text{eff}} = -\frac{3\pi G_0}{2\alpha\lambda^2} |v| \sin^2(\lambda b) + \frac{\pi_\phi^2}{2\alpha|v|}. \quad (3.14)$$

Now, the dynamics of the fundamental variables v, b, ϕ, π_ϕ is obtained by solving the system of Hamilton

equations

$$\begin{aligned} \dot{v} &= \{v, C_{\text{eff}}\} = -\frac{6\pi G_0}{\lambda\alpha} |v| \sin(\lambda b) \cos(\lambda b), \\ \dot{b} &= \{b, C_{\text{eff}}\} = \frac{3\pi G_0}{\lambda^2\alpha} \sin^2(\lambda b) + \frac{\pi_\phi^2}{\alpha v^2}, \\ \dot{\phi} &= \{\phi, C_{\text{eff}}\} = \frac{\pi_\phi}{\alpha|v|}, \quad \dot{\pi}_\phi = 0. \end{aligned} \quad (3.15)$$

These equations are not independent, since $C_{\text{eff}} = 0$ (i.e., the motions are curves in Γ lying on the constraint surface defined by imposing the vanishing of the effective Hamiltonian).

Recalling that $a = (v\alpha)^{1/3}$, one sees that $H = \dot{a}/a = \dot{v}/3v$, so using the effective Hamilton equation for \dot{v} in Eq. (3.15), and the constraint $C_{\text{eff}} = 0$, given by Eq. (3.14), one finds the effective Friedmann equation [17, 18]:

$$H^2 = \frac{8\pi G_0}{3} \rho \left(1 - \frac{\rho}{\rho_{\text{cr}}}\right), \quad (3.16)$$

where $\rho = \pi_\phi^2/(2a^6)$ and $\rho_{\text{cr}} = 3/(8\pi G_0 \gamma^2 \lambda^2) \approx 0.41 \rho_{\text{Pl}}$. Eq. (3.16) implies that the classical energy density ρ is limited to the interval $\rho_0 < \rho < \rho_{\text{cr}}$ having an upper bound at ρ_{cr} . Notice that, $\rho_0 \ll \rho_{\text{cr}}$ is the energy density of the star at the initial configuration, $t = 0$, where $\rho_0 = \pi_\phi^2/(2a_0^6)$.

On an effective geometry, in order to compare with the classical general relativity, the modified Friedmann equation (3.16) is obtained by considering an effective Newton’s constant given as $G_{\text{eff}} = G_0 (1 - \rho/\rho_{\text{cr}})$, where G is the low energy Newton’s constant. In other words, it is interesting to consider the dynamical trajectories of the collapsing system to be given by the Hubble rate $H = \dot{a}^2/a^2$, whereas the energy density is effective in a quantum geometry regime and is provided by the modified Friedmann equation (3.16). Hence, we can write

$$H^2 := \frac{8\pi G_{\text{eff}}}{3} \rho = \frac{8\pi G_0}{3} \rho_{\text{eff}}. \quad (3.17)$$

So, the effective energy density reads

$$\rho_{\text{eff}} := \rho \left(1 - \frac{\rho}{\rho_{\text{cr}}}\right). \quad (3.18)$$

We see that quantum geometry effects lead to a modification proportional to ρ^2 , which becomes important when the energy density becomes comparable to ρ_{cr} . Furthermore, in the limit $\rho \rightarrow \rho_{\text{cr}}$, the Hubble rate vanishes; the classical singularity is thus replaced by a bounce (cf. figure 1). In this sense, quantum geometry leads to a singularity resolution at the late-time evolution of the collapse. Notice that, in the limit $\rho \ll \rho_{\text{cr}}$, the standard Friedmann equation is recovered.

The classical Friedmann equation corresponds to the last relation in the classical Einstein’s field equation (2.3), given by $H^2 = F/R^3$. Consequently, the effective theory

⁴ The definitions of singularity and trapped surfaces are based on the evolution of the geodesic curves on spacetime manifolds. Therefore, in quantum theory these definitions are not valid anymore and can not be employed at the quantum level.

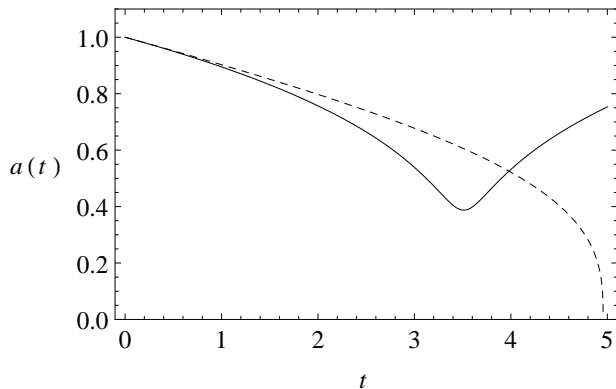


Figure 1: Behaviours of the scale factor $a(t)$ in the classical (dashed curve) and effective (solid curve) cases.

of LQC leads to a modification for the mass function in Eq. (2.3). Therefore, the mass function $F(t, r)$ can be modified on the effective geometry as

$$F_{\text{eff}} = \frac{8\pi G_0}{3} \rho_{\text{eff}} R^3. \quad (3.19)$$

The phase space trajectories are considered classical, whereas the matter content is assumed to be quantized. In the classical limit, as $\rho_{\text{eff}} \rightarrow \rho$, the effective mass function reduces to the classical $F = (8\pi G_0/3)\rho R^3$ or $F = (8\pi G_0/3\sqrt{2})p_\phi^2 r_b^3 \sqrt{\rho}$. Eq. (3.19) shows that, as $\rho_0 < \rho < \rho_{\text{cr}}$ on the interior quantum geometry, the effective mass function remains finite during the collapse. It is convenient to rewrite the Eq. (3.19) as

$$F_{\text{eff}} = F \left(1 - \frac{F^2}{F_{\text{cr}}^2} \right), \quad (3.20)$$

where $F_{\text{cr}} := \frac{8\pi G_0}{3} \rho_{\text{cr}} R_{\text{cr}}^3 = \frac{8\pi G_0}{3\sqrt{2}} \pi_\phi^2 r_b^3 \sqrt{\rho_{\text{cr}}}$, is a constant. So that, we can also write

$$\frac{F^2}{F_{\text{cr}}^2} = \frac{\rho}{\rho_{\text{cr}}}. \quad (3.21)$$

Eq. (3.20) shows that, mass function F changes in the interval $F_0 < F < F_{\text{cr}}$ along with the collapse dynamical evolution. Nevertheless, if on the initial condition the mass function starts with $F < R$, then as the scale factor decreases, the mass function also decreases and becomes zero at the bounce; consequently, there will be no trapped surfaces forming. This result is consistent with the case presented in the previous section.

IV. EFFECTIVE DYNAMICS OF HORIZONS AND EXTERIOR GEOMETRY

To discuss the dynamics of the trapped region in the perspective of the effective dynamics scenario, particular importance is played by the function Θ_b given by Eq.

(2.9). Therein, by replacing H with the effective Friedmann equation (3.16) we get

$$\Theta_b = \frac{4\pi G_0}{3} \rho \left(1 - \frac{\rho}{\rho_{\text{cr}}} \right) - \frac{1}{2a^2 r_b^2}. \quad (4.1)$$

We can study the behaviour of the effective Θ_b as a function of scale factor a . Since the cloud is initially untrapped, at the initial space slice (i.e., for large a) $\Theta_b(t=0)$ is negative. Then, as the collapse proceeds (i.e., a decreasing towards the collapse center), the second term always increases, whereas the first term (the Hubble rate) increases, whose effect being dominant until the energy density reaches to $\rho = 0.4\rho_{\text{cr}}$ (cf. figure 2); at this point Θ_b reaches its maximum, Θ_{max} :

$$\Theta_{\text{max}} = 0.32\pi G_0 \rho_{\text{cr}} - \left(\frac{\rho_{\text{cr}}}{10\pi_\phi^2 r_b^6} \right)^{\frac{1}{3}}. \quad (4.2)$$

Figure 2 shows the behaviour of Θ_b against the scale factor a for the different choices of the initial conditions. Solid curve represents the trajectories provided by the effective dynamics gravitational collapse. Dashed curve shows the classical trajectories which coincides the effective equations for large values of a . Equation of apparent horizon on the effective geometry can be obtained by setting $\Theta_b = 0$. Therefore, depending on the initial conditions, in particular on the choice of the r_b , three cases can be evaluated. The left plot (solid curve) indicates a condition in which an untrapped interior spacetime without any horizon forming, whereas two others show trapped regions; the middle and the right figures correspond to one and two horizons formation, respectively. In addition, denoting by dashed curves, always only one horizon can form classically.

The scale factor a_{max} , corresponding to Θ_{max} reads

$$a_{\text{max}} = \left(\frac{\pi_\phi^2}{0.8\rho_{\text{cr}}} \right)^{\frac{1}{6}}. \quad (4.3)$$

Notice that this value is independent of r_b , so it is the same for any shell. The minimum value of the scale factor is fixed by the requirement that the Hubble rate vanishes, i.e., $\rho = \rho_{\text{cr}}$ where the collapse hits a bounce as its final state. Using the relation $\rho = \pi_\phi^2/(2a^6)$, one easily computes the scale factor a_{cr} as

$$a_{\text{cr}} = \left(\frac{\pi_\phi^2}{2\rho_{\text{cr}}} \right)^{\frac{1}{6}} = (2.5)^{1/6} a_{\text{max}}. \quad (4.4)$$

The modified Friedmann equation (3.16) allows to compute the maximum speed of the collapse $|\dot{a}|_{\text{max}}$, corresponding to the scale factor a_{max} :

$$|\dot{a}|_{\text{max}} = 0.32\pi G_0 \left(\frac{\pi_\phi \rho_{\text{cr}}}{10} \right)^{\frac{2}{3}}. \quad (4.5)$$

On the other hand, by Eq. (2.8) we can determine the speed of the collapse, $|\dot{a}|_{\text{AH}}$, at which horizons do form,

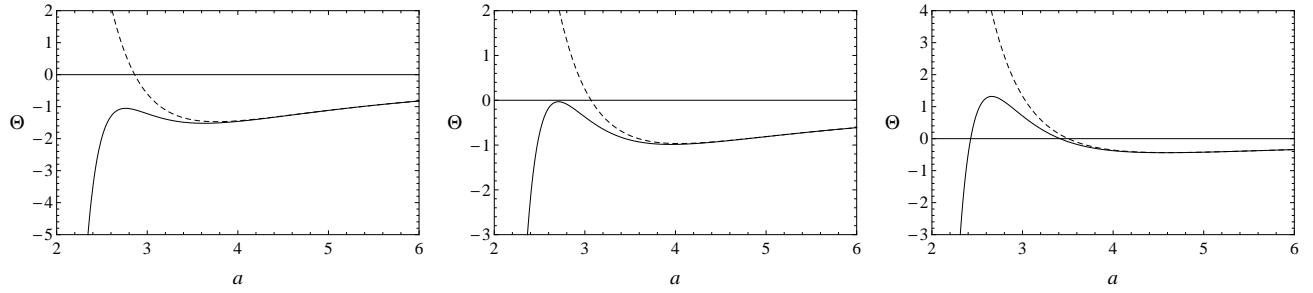


Figure 2: Behaviours of $\Theta_b(a)$ in the classical (dashed curve) and quantum (solid curve) cases, for different values of r_b (from left to right: $r_b < r_*$, $r_b = r_*$ and $r_b > r_*$). Where r_* defines the threshold radius.

by simply setting $\Theta = 0$. Then, we get $\dot{R}^2 = 1$, which for the boundary shell, gives

$$|\dot{a}|_{\text{AH}} = \frac{1}{r_b}. \quad (4.6)$$

When the speed of collapse, $|\dot{a}|$, reaches the value $1/r_b$, then an apparent horizon forms. Thus, if the maximum speed $|\dot{a}|_{\text{max}}$ is lower than the critical speed $|\dot{a}|_{\text{AH}}$, no horizon can form. Let us introduce a *threshold radius* r_* , defined by

$$r_* := \frac{1}{|\dot{a}|_{\text{max}}}. \quad (4.7)$$

We see that, if $r_b < r_*$, then no horizon can form at any stage of the collapse. The case $r_b = r_*$ corresponds to the formation of a dynamical horizon at the boundary of the two spacetime regions [24]. Finally, for the case $r_b > r_*$ two horizons will form, one inside and the other outside of the collapsing matter (cf. figure 3) [11].

The total mass m inside the collapsing star is given by the integral of the energy density on a sphere of radius $R_b = ar_b$:

$$m = \int a^3 \rho_\phi d^3x = \frac{2\pi}{3} \frac{\pi_\phi^2}{a_0^3} r_b^3. \quad (4.8)$$

Thus, we can translate the condition $r_b \geq r_*$, on the radius, into a condition on the mass of star. Let us define a m_* to be the mass of a star with the shell radius r_* :

$$m_* := \frac{2\pi}{3} \frac{\pi_\phi^2}{a_0^3} r_*^3. \quad (4.9)$$

Therefore, in order for trapped surfaces to form, the mass, m , of a collapsing star must be bigger than the threshold mass m_* . In other words, the minimum mass of the final black hole must be bigger than the threshold mass, $m \geq m_*$. This inequality cannot be satisfied in the case of micro black holes, since in that case m is extremely small. Thus, for such black holes (which are expected to form in the primordial universe, as well as in LHC), trapped surfaces never form, and so information about the interior structure can travel outside, to the distant observer.

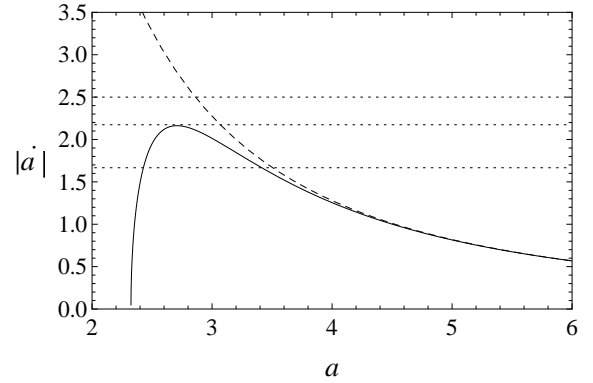


Figure 3: The speed of collapse, $|\dot{a}|$, with respect to the scale factor a , in classical (dashed curve) and semiclassical (solid curve) regimes.

So far we have analysed the interior spacetime of the collapse in the presence of the quantum gravity effects. This quantum effects is expected to be carried out to the exterior geometry by using the matching conditions on the boundary r_b of two regions. In the following we will focus on the main physical consequences that can emerge from this scenario in order to predict the possible exterior geometry for the collapse. To this aim, we will consider two cases: First, we will investigate the fate of the collapsing star whose mass is less than the threshold mass m_* . Secondly, we will study the exterior geometry for the case in which the mass of the star is bigger than the threshold mass whereby a black hole can form.

A. Outward flux of energy

For a collapsing star whose mass is less than m_* , we study the mass loss using the quantum corrections to the interior spacetime due to the effective theory we discussed before.

Let us designate the mass function at scales $v \gg 1$, i.e, in the classical regime, as $F = (8\pi G_0/3)\rho R^3$, whereas for $v \approx \alpha^{-1}$ (in the quantum geometry regime) we use F_{eff} given by Eq. (3.20). Then, the mass loss, $\Delta F/F$, is

provided by the following expression:

$$\frac{\Delta F}{F} = 1 - \frac{F_{\text{eff}}}{F} = \frac{\rho}{\rho_{\text{cr}}} = \frac{F^2}{F_{\text{cr}}^2}. \quad (4.10)$$

This equation can be interpreted as follows: If $\rho \ll \rho_{\text{cr}}$, i.e. at the classical limit, then $\Delta F \approx 0$. However, when the energy density $\rho \rightarrow \rho_{\text{cr}}$ at the critical value (bouncing point) in the semiclassical limit, we have $\Delta F/F \rightarrow 1$; this means that when the collapsing cloud approaches the critical energy density, an outward flux of energy will start near the bounce.

It is worthwhile to mention that, within an inverse triad correction of collapsing system (with a scalar field [10], or a tachyon field [13], as matter source), the (quantum) modified energy density decreases as collapse evolves. Whence, as the collapsing cloud approaches the center (with a vanishing scale factor, where the classical singularity is located) the energy density reaches its minimum value, whereas the mass loss tends to one. In the model herein, as the gravitational collapse proceeds in the quantum regime, the energy density increases and reaches its maximum value ρ_{cr} at the bounce (with a finite non-zero volume). In this process, however, the mass loss tends to unit earlier, and before that the collapse reaches to the center of star.

Let us assume that the energy density flux is measured locally by an observer with a four-velocity vector ξ^μ . Then, the energy flux and the radiation energy density are measured in this local frame and given by $\sigma \equiv T_{\mu\nu}\xi^\mu\xi^\nu$. Furthermore, the total luminosity for a radially moving observer with radial velocity $\vartheta \equiv \xi^r = \frac{dr_v}{dt}$ at the radius r_v , is given by $L(v) = 4\pi r_v^2 \sigma$ [25], so that:

$$L(v) = -\frac{1}{(\gamma + \vartheta)^2} \frac{dM(v)}{dv}, \quad (4.11)$$

where $\gamma = (1 + \vartheta^2 - 2M(v)/r_v)^{-1}$. As long as $dM/dv \leq 0$, the total luminosity of the energy flux is positive; this indicates that there exist an energy flux radiated away from interior spacetime and reaching the distant observer. Moreover, from $\xi_\mu \xi^\mu = -1$ and $\xi^\theta = \xi^\varphi = 0$, we have

$$\frac{dv}{dt} = \xi^v = \frac{1}{\gamma + \vartheta}. \quad (4.12)$$

Now, using Eqs. (4.11) and (4.12), and substituting $M(v)$ with $M = F(t, r_b)/2$ at the boundary Σ we obtain,

$$L(v) = -\left(\frac{\dot{F}_{\text{eff}}}{2}\right) \frac{1}{(\gamma + \vartheta)}. \quad (4.13)$$

For an observer at rest ($\vartheta = 0$) and infinitely distant ($r_v \rightarrow \infty$), the total luminosity of the energy flux, L_∞ , can be obtained by taking the limit of Eq. (4.13):

$$L_\infty(t) = -\left(\frac{\dot{F}_{\text{eff}}}{2}\right). \quad (4.14)$$

Using Eq. (3.19) in Eq. (4.14), we can estimate the time variation of the mass function as

$$\begin{aligned} \dot{F}_{\text{eff}} &= -8\pi G_0 H R^3 \rho \left(1 - 3\frac{\rho}{\rho_{\text{cr}}}\right) \\ &= 8\pi G_0 R^3 \left(\frac{8\pi G_0}{3}\rho_{\text{eff}}\right)^{\frac{1}{2}} \rho \left(1 - 3\frac{\rho}{\rho_{\text{cr}}}\right). \end{aligned} \quad (4.15)$$

The classical Raychaudhuri equation, $\frac{\ddot{a}}{a} = -\frac{4\pi G_0}{3}(\rho + 3p)$, in effective theory can be modified from Hamilton's equation as follows:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_0}{3} \left[-2\rho \left(1 - \frac{\rho}{\rho_{\text{cr}}}\right) + 6\rho \left(1 - 2\frac{\rho}{\rho_{\text{cr}}}\right) \right]. \quad (4.16)$$

So that, by rewriting the modified Raychaudhuri equation as $\frac{\ddot{a}}{a} = -\frac{4\pi G_0}{3}(\rho_{\text{eff}} + 3p_{\text{eff}})$, we can define the effective pressure as

$$p_{\text{eff}} := \rho \left(1 - 3\frac{\rho}{\rho_{\text{cr}}}\right). \quad (4.17)$$

Inserting Eq. (4.17) in Eq. (4.15), the time derivative of the mass function can also be expressed as

$$\dot{F}_{\text{eff}} = 8\pi G_0 R^3 \left(\frac{8\pi G_0}{3}\rho_{\text{eff}}\right)^{\frac{1}{2}} p_{\text{eff}}. \quad (4.18)$$

The classical limit of Eq. (4.18) is

$$\dot{F} = 8\pi G_0 p R^3 \left(\frac{8\pi G_0}{3}\rho\right)^{\frac{1}{2}} = -8\pi G_0 p R^2 \dot{R}, \quad (4.19)$$

as expected from Eq. (2.3). Eq. (4.18) shows that in last stage of the gravitational collapse, the mass function decreases when the effective pressure becomes negative. In contrast, the classical system shows a continuous increase of the mass function, with an increasing positive pressure, pointing to a black hole end state. In figure 4 we have a graphical representation of Eq. (4.18) during evolution of the collapse. Therein, we have that the effective mass function's time derivative, being initially positive (like its classical counterpart), starts to decrease until it becomes negative and then vanishes at the bounce. At this stage, as $L_\infty \approx -\dot{F}_{\text{eff}}$, the luminosity positiveness is related to the mass loss occurring near the bounce. The existence of a negative \dot{F}_{eff} , with some substantial variation in the vicinity of the bounce, points to the existence of an energy flux radiated away from the interior spacetime and reaching the distant observer [25].

Figure 5 presents the behaviour of the effective pressure (4.17) conveniently scaled with the critical density ρ_{cr} . Therein we should stress that the effective pressure becomes super negative near the bounce and takes the value $p_{\text{eff}}(\rho_{\text{cr}}) = -2\rho_{\text{cr}}$ at the bounce. This verifies the general understanding derived from homogeneous isotropic models indicating that, the singularity resolution is associated with the violation of energy conditions. This suggests that quantum gravity provides a repulsive force at very short distances [26].

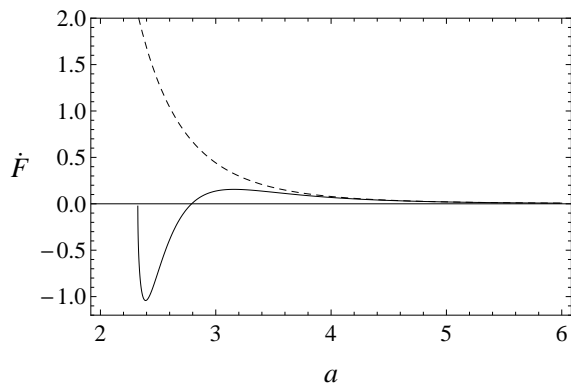


Figure 4: Behaviour of the time derivative of the mass function, \dot{F} , in the effective quantum dynamics regime (from Eq. (4.18)) for the value of parameters $G_0 = c_{\text{light}} = 1$, and $\pi_\phi = 10\,000$. The full line is for effective dynamics, whereas the dashed line is for the classical counterpart.

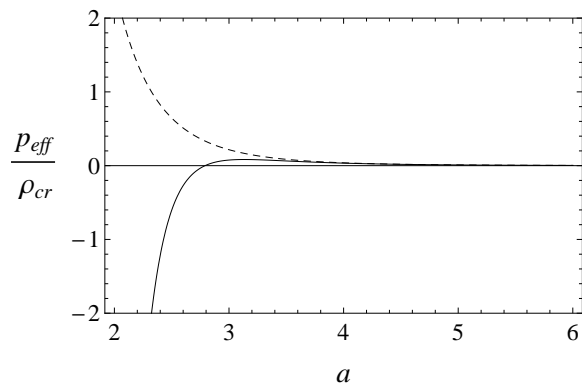


Figure 5: Behaviour of the effective pressure (from Eq. (4.17)) for the value of parameters $G_0 = c_{\text{light}} = 1$, and $\pi_\phi = 10\,000$. At the bounce we have $p_{\text{eff}}(\rho_{\text{cr}}) = -2\rho_{\text{cr}}$. The full line is for effective dynamics, whereas the dashed line is for the classical counterpart.

B. Non-singular black hole formation

If the initial mass of the collapsing star is bigger than the threshold mass m_* , given by Eq. (4.9), then a black hole would form at the collapse final stage. In this section we will analyse a possible prediction for the exterior geometry of the collapsing system in this case.

The total mass measured by an asymptotic observer is given by $m_{\text{ext}} = m_M + m_\phi$, where m_M is the total mass in the generalized Vaidya region, and $m_\phi = \int \rho dV$ is the interior mass related the scalar field ϕ . Since the matter related m_M is not specified in the exterior Vaidya geometry in our model, we just focus on a qualitative analysis of behaviour of the horizon close to the matter shells.

From the matching conditions (2.11)-(2.14), we can get the information regarding the behaviour of trapping horizon in the exterior region. Indeed, when the relation

$2M(v, r_v)G_0 = r_v$ is satisfied at the boundary, trapped surfaces will form in the exterior region close to the matter shells. In classical geometry, the boundary function, $\mathcal{F} = (1 - 2M(v)G_0/r_v)$, becomes negative in the trapped region and vanishes on the apparent horizon. Therefore, the equation for event horizon is given at the boundary of the collapsing body by $\mathcal{F}|_\Sigma = 0$. Nevertheless, in semi-classical regime, the boundary function is expected to be modified by employing the matching conditions due to the fact that the interior spacetime was modified by the quantum gravity effects.

Using the conditions (2.11) and (2.13), the classical function \mathcal{F} can be written as

$$\mathcal{F}|_\Sigma = 1 - \frac{F}{R}, \quad (4.20)$$

where we have used $2M(v, r_v)G_0/r_v = F(t)/R(t)$ at the boundary surface Σ with $r = r_b$. In semiclassical regime, the mass function is modified as Eq. (3.19), whereby the boundary function (4.20) is modified as

$$\mathcal{F}_{\text{eff}} = 1 - \frac{F}{R} \left(1 - \frac{F^2}{F_{\text{cr}}^2} \right), \quad (4.21)$$

at the boundary shell r_b . Eq. (4.21) shows that the quantum gravity effects leads to a modification of the boundary function by a cubic term F^3 .

By substituting the classical mass function with $F = (8\pi G_0/3)\rho R^3$, we can rewrite the Eq. (4.21) as

$$\mathcal{F}_{\text{eff}}(R) = 1 - \frac{A}{R^4} \left(1 - \frac{B}{R^6} \right), \quad (4.22)$$

where $A \equiv (4\pi G_0/3)\pi_\phi^2 r_b^6$, and $B \equiv \pi_\phi^2 r_b^6/(2\rho_{\text{cr}})$ are constants. Eq. (4.22) represents a non-singular, exotic black hole geometry. Notice that, this function for large values of R tends to the classical limit:

$$\mathcal{F}(R) = 1 - \frac{A}{R^4}, \quad (4.23)$$

which represents a classical singular black hole geometry [12]. In this case, the apparent horizon could form when \mathcal{F} in Eq. (4.23) vanishes; equation of this horizon which intersects with the matching surface is given by $a(t) = A^{1/4}/r_b$. Furthermore, as collapse tends to the singularity as $R \rightarrow 0$, the boundary function diverges as $\mathcal{F} \rightarrow -\infty$; the classical singularity is covered by the black hole horizon. In addition, as we expected, in the presence of a nonzero matter pressure (of the massless scalar field) at the boundary, the (homogeneous) interior spacetime can not be matched with an empty (inhomogeneous) Schwarzschild exterior [11].

V. CONCLUSIONS AND DISCUSSION

We considered a spherically symmetric and homogeneous spacetime for a collapsing system, with a massless

scalar field as matter content. The homogeneous interior is matched with an exterior Vaidya geometry.

We subsequently studied the interior spacetime within the effective theory of loop quantum gravity, provided by a holonomy correction to the dynamics of the system [18]. It was shown that loop quantum effects removes the classical singularity that arises at the end state of the collapse, and replaces it by a quantum bounce. Furthermore, we investigated the evolution of the trapped surfaces emerging from the semiclassical interior spacetime. The physical modifications related to the semiclassical regime provided three cases for the trapped surfaces formation, depending on the initial conditions of the collapsing star. In particular, our solutions showed that, if the initial mass of the collapsing cloud is less than a threshold mass, no horizon forms during the collapse, whereas for the mass equal and larger than the threshold mass, one and two horizons form, respectively. It is worthy to mention that, this scenario is qualitatively similar to the model previously predicted from an inverse triad correction [11].

The interior semiclassical effects is carried out to the exterior geometry by the matching conditions on the boundary of two regions. Hence, an effective geometry emerged for the exterior metric which describes the physical consequences for the late-time evolution of the collapse. In the case in which no horizon forms, we have showed that, as the collapse evolves, the energy density and mass function increase towards the maxima ρ_{cr} and F_{cr} , respectively. However, this energy growth is accompanied by the effective mass loss, induced by the quantum geometry regime. Therefore, in the late-time stages of the collapse, it gives rise to an outward flux of energy from the interior quantum spacetime to the distant observer. Similar results was discussed for a scalar field collapse, where an inverse triad modification was employed [10]. However, the mass loss obtained therein, was characterized by a reduction of the energy density and mass function towards the center of the star. In the other case in which one or two horizon form, the exterior geometry obtained predicts an exotic black hole formation that is different than the Schwarzschild one.

It has been proposed that gamma-ray bursts may occur at the final state of the collapse [27]. In our model herein, although the effective theory is not accurate in a deep Planck regime, there might be some stages of the gravitational collapse where the effective quantum geometry becomes relevant. Therefore, from such a scenario it could be observed some of those outward flux of energy from gamma-ray bursters.

Modification to the effective mass function through the quantum geometry effect at the boundary of two re-

gions, results in an effective exterior Vaidya geometry. More precisely, from Eq. (3.20) we have that $F_{\text{eff}} = F(G_{\text{eff}}/G_0)$. Therefore, using Eq. (2.13), we get $F_{\text{eff}} = 2MG_{\text{eff}}$, so that

$$M_{\text{eff}}G_0 = MG_{\text{eff}}. \quad (5.1)$$

Consequently, the Vaidya metric (2.10) takes the improved form:

$$\begin{aligned} \tilde{g}_{\mu\nu}^+ dx^\mu dx^\nu = & -(1 - 2M(v)G_{\text{eff}}/r_v) dv^2 \\ & - 2dvdr_v + r_v^2 d\Omega^2. \end{aligned} \quad (5.2)$$

The improved Vaidya solution (5.2) shows that the effective quantum (interior) spacetime is manifest through a modification to the Newton constant G_0 as $G_0(1 - \rho/\rho_{cr})$. Similar scenario was presented in recent works [28–30]. Therein, it was shown that, in situations where the interior spacetime undergoes a transition from positive to negative pressures, a quantum improvement to the Vaidya outgoing solution should be implemented. Nevertheless, this improved outgoing Vaidya solution, results in a modification of the Newton constant as $G(r_v)$ being a scale dependent function [28]. The redefinition of the Newton constant in this context was introduced to model quantum effects and avoid the problem of unbound back scattered radiation [28].

The qualitative picture that emerges from our toy model was influenced by a homogeneous interior spacetime, however, in a realistic collapsing system, one has to employ a more general (inhomogeneous) setting. Nonetheless, in the full dynamics of states in LQG, it seems that the ‘energy’ in the inhomogeneities is sufficiently small even in the Planck era, where the inhomogeneous modes will be affected by the quantum geometry effects of the dominant homogeneous modes [31]. In this case, the inhomogeneous modes would be too weak for their own quantum geometry effects to be important. Therefore, the effective theory, as an approximation of the improved dynamics LQC, can be a good phenomenological approach to study the gravitational collapse.

VI. ACKNOWLEDGEMENT

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